B.SC. FOURTH SEMESTER (PROGRAMME) EXAMINATIONS, 2021

Subject: Mathematics **Course ID: 42110**

Course Title: Graph Theory **Course Code: SP/MTH/404/SEC-2**

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning.

1. **Answer *any five* of the following questions:** **(2×5=10)**

a) Define graph and digraph.

b) Define graph isomorphism with example.

c) Define Eulerian graph and Hamiltonian graph.

d) Show that a complete graph with n number of vertices has $\frac{n(n-1)}{2}$ number of edges.

e) Draw simple graphs of three and four vertices, and two and five edges, respectively.

f) Show that every acyclic graph is a simple graph.

g) How many edges does the graph $K_{3,6}$ contain?

h) Draw a graph with the following matrix as adjacent matrix

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

2. **Answer *any four of* the following questions:** **(5×4=20)**

a) (i) If G is a simple graph of order n with minimum vertex degree $\geq \frac{n-1}{2}$, then show that G is connected.

(ii) If a graph contains exactly two vertices of odd degree then prove that there exists a path between these two vertices. 3+2 = 5

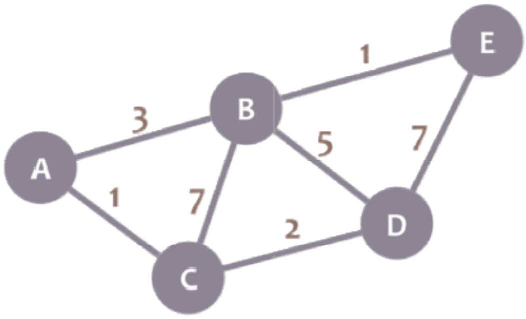
b) (i) If G is a graph in which the degree of each vertex is at least 2, then show that G contains a cycle.

(ii) Let Gbe a graph with n number of vertices and $n-1$ number of edges. Prove that G has either a pendant vertex or an isolated vertex. 3+2 = 5

c) Define tree. Show that every tree has atleast two leaves. 1+4=5

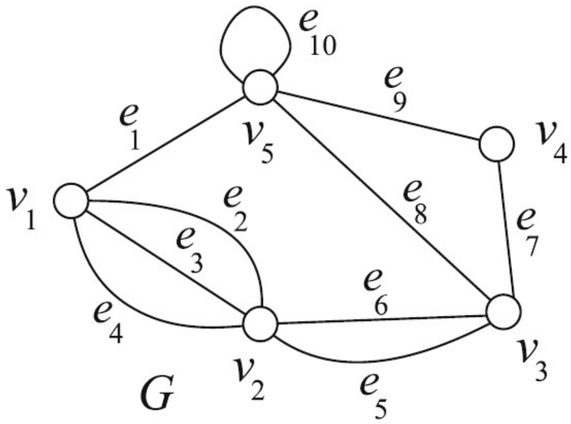
d) Draw a graph with five vertices v_1, v_2, v_3, v_4, v_5 such that $deg\ v_1 = 3, v_2$ is an odd degree vertex, $deg\ v_3 = 2, v_4$and v_5 are adjacent. 5

e) Using Dijkstra's algorithm, find the shortest path from the node C to E in the following graph



5

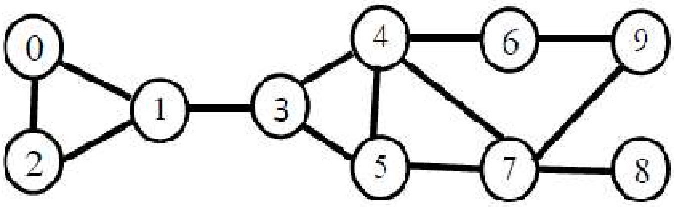
f) Define the adjacency matrix and incidence matrix of a graph. Find out the adjacency matrix and incidence matrix of the following graph G. 2+3=5



3. Answer *any one* of the following questions: (10×1=10)

a) (i) Prove that a graph T is a tree if and only if each pair of vertices of T are connected by a unique path. 5+1+4

(ii) Define a spanning tree of a graph. Find a spanning tree of the following graph.



b) (i) Let G be a nonempty graph with order n (> 2). If G is bipartite, then prove that the length of any cycle of the graph is even.

(ii) Show that a simple graph (order ≥2) has atleast two vertices of the same degree. 5+5

***